EFFECT OF A GRAVITATIONAL FIELD ON THE STRATIFICATION OF SUSPENSIONS IN VERTICAL FLOWS

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A structuring effect is seen in flows of suspensions, with this effect having a large impact on the hydraulic properties of such flows. The numerous experiments which have substantiated the radial migration of spherical particles were discussed in [1]. These experiments show that in a Poiseuille flow of suspensions of equal density, particles are displaced toward the center of the channel. In the case of the ascending motion of a fluid with relatively heavy particles and the descending motion of a fluid with relatively light particles, the latter tend to accumulate in the central region of the flow and form a tightly packed core. If the suspension of heavy particles moves downward or the suspension of light particles moves upward, then the opposite pattern is seen: the particles accumulate near the walls of the channel. A similar situation exists in ascending flows of a mixture of fluids with small bubbles. In this case, an increase in gas content is seen in the wall region [2, 3]. Such structuring in vertical flows is due to the effect of an inertial buoyant force on the particles in the transverse direction [4]. A model which makes it possible to explain the existence of nonuniform concentration profiles in equidense suspensions was presented in [5]. It was assumed in the model that a stationary distribution of concentration is attained because the transverse particle flow due to buoyancy is counterbalanced by an oppositely directed diffusion flow. The latter flow is described by introducing a thermodynamic force which acts on the particles. This force is found from the condition of equality of the diffusion flow to the convective flow created by the force. In the present study, we apply this approach to vertical flows of suspensions of different densities in a gravitational field.

We will examine a monodisperse suspension of fine spherical particles of radius a and density d_1. The particles are placed in a fluid of density d_0. For the sake of definiteness, we assume that, due to the smallness of the particles, only isotropic Brownian motion actually contributes to diffusion.

The following expressions [6] give the longitudinal components of the momentum conservation equation for a suspension and its disperse phase moving vertically inside a pipe in a gravitational field

$$-\frac{\partial p}{\partial x}+\mu_0\frac{1}{y}\frac{d}{dy}\left[M(\rho)\,y\frac{dv}{dy}\right]+dg=0; \tag{1}$$

$$\beta M(\rho)\,u+(1-\rho)(d_1-d_0)\,g=\frac{3}{4}\mu_0\frac{1}{y}\frac{d}{dy}\left[M(\rho)\,y\frac{dv}{dy}\right],\quad \beta=9\mu_0/2a^2,\ d=d_0(1-\rho)+\rho d_1. \tag{2}$$

Here x and y are longitudinal and radial coordinates; u = v − w is phase slit velocity; v and w are the mean velocities of the fluid and the particles; p is pressure; μ_0 is the viscosity of the pure fluid; g is acceleration due to gravity; $M(\rho)$ is an increasing function of the volume concentration of the disperse phase ρ.

Equation (2) was derived on the basis of allowance for the viscous Stokes force f_S, the Faxon force f_F, the gravitational force, and buoyancy. The expressions we use for f_S and f_F are taken from [5]:

$$f_S=\frac{9}{2}\rho\frac{\mu_0}{a^2}M(\rho)\,u,\quad f_F=\frac{3}{4}\rho\mu_0\frac{1}{y}\frac{d}{dy}\left[M(\rho)\,y\frac{dv}{dy}\right].$$

For the transverse force, we use an expression obtained in [4]:

$$f_M=\rho\frac{3\cdot 6{,}46}{4\pi a}d_0\left[\nu_0 M(\rho)\left|\frac{dv}{dy}\right|\right]^{1/2}u\,\mathrm{sign}\left\{\frac{dv}{dy}\right\} \tag{3}$$

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(ν_0 is the kinematic viscosity of the pure fluid). This force causes the particles to migrate in the transverse direction, thus helping to form a nonuniform concentration profile. In accordance with [5], the transverse component of the momentum conservation equation for the disperse phase can be written on the basis of the condition of equality of the force (3) to the thermodynamic force

$$f_T = -\frac{3\rho}{4\pi a^3}\left[\frac{\partial\varphi}{\partial\rho}\right]_{p,T}\frac{d\rho}{dy}, \tag{4}$$

where φ is the chemical potential of the particles: $\varphi = \text{const} + kTF(\rho)$, $F(\rho) = \ln\rho - \rho + \rho\frac{8-5\rho}{(1-\rho)^2}$, while differentiation is performed with constant pressure and temperature.

Then introducing dimensionless variables and parameters:

$$\xi = \frac{y}{R},\ V = \frac{\mu_0 v}{R^2 P},\ \alpha = \frac{g}{P}d_0,\ \gamma = \frac{g}{P}d_1$$

(R is the tube radius, $P = -\partial p/\partial x$), Eq. (1) can be transformed as follows:

$$\frac{1}{\xi}\frac{d}{d\xi}\left[M(\rho)\,\xi\frac{dV}{d\xi}\right] + \alpha(1-\rho) + \gamma\rho + 1 = 0. \tag{5}$$

Taking into account the expression for slip velocity which follows from (2),

$$u = \left\{\frac{3}{4}\frac{1}{\xi}\frac{d}{d\xi}\left[M(\rho)\,\xi\frac{dV}{d\xi}\right]P - (1-\rho)(d_1-d_0)\,g\right\}[\beta M(\rho)]^{-1}, \tag{6}$$

we represent the transverse component of the momentum conservation equation of the disperse phase as

$$M(\rho)\frac{dF}{d\rho}\frac{d\rho}{d\xi} = \Gamma\left[M(\rho)\left|\frac{dV}{d\xi}\right|\right]^{1/2}\left\{\frac{1}{\xi}\frac{d}{d\xi}\left[M(\rho)\,\xi\frac{dV}{d\xi}\right]A - \frac{4}{3}(1-\rho)(\gamma-\alpha)\right\},\ \Gamma = \frac{6{,}46}{6}\frac{a^4(PR)^{3/2}}{d_0^{1/2}\nu_0 kT},\ A = \text{sign}\left\{\frac{dV}{d\xi}\right\}. \tag{7}$$

The boundary conditions for (5) and (7) follow from considerations of symmetry and adhesion on the channel walls:

$$V = 0 \text{ for } \xi = 1,\ dV/d\xi = 0 \text{ (or } d\rho/d\xi = 0) \text{ for } \xi = 0. \tag{8}$$

We obtain yet another condition if we know either $\langle\rho\rangle$ (particle concentration averaged over the cross section) or ρ_f (mean-flow rate concentration):

$$\langle\rho\rangle = 2\int_0^1 \rho(\xi)\,\xi\,d\xi,\ \rho_f = \int_0^1 \rho(\xi)\,V(\xi)\,\xi\,d\xi\left\{\int_0^1 V(\xi)\,\xi\,d\xi\right\}^{-1}. \tag{9}$$

With allowance for (5), Eq. (7) takes the form of

$$M(\rho)\frac{dF}{d\rho}\frac{d\rho}{d\xi} = -\Gamma\left|M(\rho)\frac{dV}{d\xi}\right|^{1/2}\left\{1 + \frac{4}{3}\gamma - \frac{\alpha}{3} - \frac{1}{3}(\gamma-\alpha)\rho\right\}. \tag{10}$$

Thus, we have obtained system (5) and (10) to determine the unknown functions $V(\xi)$ and $\rho(\xi)$. As in [4], we used the approximation formula $M(\rho) = (1-\rho)^{-5/2}$.

A numerical analysis of these equations was performed for two different situations. In the first case, we assumed that the effect of the Faxen force was negligible compared to the effects of gravity and bouyancy. This is valid for moderate pressure gradients. Then Eq. (5) remains as before, and Eq. (10) is simplified as follows:

$$M(\rho)\frac{dF}{d\rho}\frac{d\rho}{d\xi} = -\Gamma\left|M(\rho)\frac{dV}{d\xi}\right|^{1/2}\{\gamma-\alpha\}(1-\rho).$$

Figure 1 shows different regions of structuring of the particles of the suspension in the space of the parameters α and γ. We used the condition $(1-\rho)\alpha + \rho\gamma = 1$ to find the boundary of the parameter regions which characterize the motion of the suspension upwards or downwards (this line is represented by dots). The relation $(1-\rho)\alpha + \rho\gamma < 1$ corresponds to upward motion of the suspension. The analogous inequality with the opposite sign corresponds to downward motion of the suspension. Proceeding on the basis of the absence of viscous structuring with a trivial phase-slip velocity U, we obtain the equation ($\alpha = \gamma$) of the dashed straight line in the figure; the parameters lying on this line correspond to a uniform distribution of particle concentration in the flow and, thus, to a strictly parabolic (Poiseuille) profile of fluid velocity. The values of α and γ taken from region I correspond to

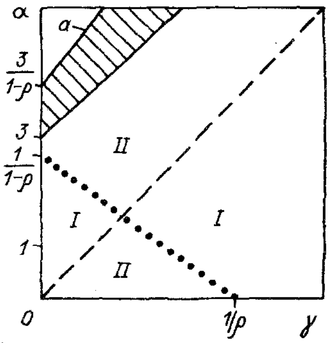

Fig. 1

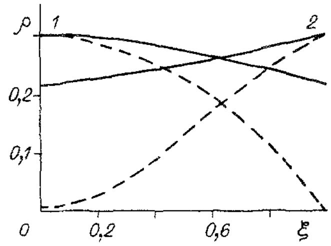

Fig. 2

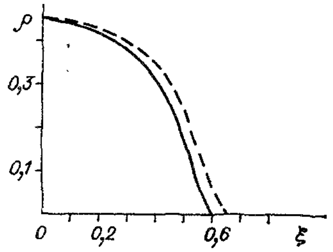

Fig. 3

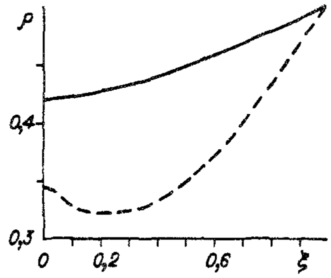

Fig. 4

TABLE 1

ρ_0	Γ							
	100				250			
	γ	α	γ	α	γ	α	γ	α
	0,2	0,1	0,1	0,2	0,2	0,1	0,1	0,2
	$\langle\rho\rangle$	ρ_f	$\langle\rho\rangle$	ρ_f	$\langle\rho\rangle$	ρ_f	$\langle\rho\rangle$	ρ_f
0,15	0,0605	0,0802	0,0325	0,0464	0,0206	0,0334	0,0004	0,0054
0,3	0,259	0,270	0,240	0,250	0,153	0,196	0,070	0,127
0,45	0,441	0,443	0,438	0,440	0,429	0,434	0,417	0,423
0,6	0,5989	0,5992	0,5985	0,5987	0,5973	0,5979	0,5963	0,5968

migration of particles toward the walls of the tube. The parameters from region II describe migration of the particles toward the center of the flow. We constructed profiles of concentration $\rho(\xi)$ for the case of upward motion of the suspension. These profiles are shown in Fig. 2 with the condition of displacement of the particles toward the center (curve 1) or toward the tube walls (curves 2). The solid lines represent graphs of the function $\rho(\xi)$ at $\Gamma = 100$, while the dashed lines show the same for $\Gamma = 250$. In these calculations, we assumed that we knew the concentration of the disperse phase ρ_0 on the channel axis (with migration of the particles toward the center of the flow) or near the tube wall (in the opposite situation).

We then used Eq. (9) to determine the mean concentrations $\langle\rho\rangle$ and ρ_f for different ρ_0, α, γ, and Γ. The results of these calculations are shown in Table 1.

A similar analysis was performed with the condition that the particles of the disperse phase are acted upon not only by gravity and buoyancy, but also by Faxen forces. This situation prevails when there are very large pressure gradients in a suspension of a relatively light fluid and particles. The fluid should also have a high viscosity. Then the functions $V(\xi)$ and $\rho(\xi)$ will be determined from Eqs. (10) and (5). As in the case of the absence of the Faxen force, we found the structure-formation regions in the space of the parameters α and γ. These regions are shown in Fig. 1. As before, in the present situation, the dotted line is the line separating the parameter regions corresponding to upward or downward motion of the suspension. Line a was found from the condition of equality of phase-slip velocity to zero. For example, the coordinates of points lying below this line and above the dotted line characterize migration of the particles toward the channel walls. It is evident from

TABLE 2

Γ	ρ_0	γ = 0, α = 0: ⟨ρ⟩	γ = 0, α = 0: ρ_f	γ = 0,2, α = 0,1: ⟨ρ⟩	γ = 0,2, α = 0,1: ρ_f
100	0,15	0,0021	0,0042	0,0006	0,0029
	0,3	0,016	0,030	0,011	0,029
	0,45	0,121	0,173	0,096	0,151
	0,6	0,563	0,571	0,561	0,570

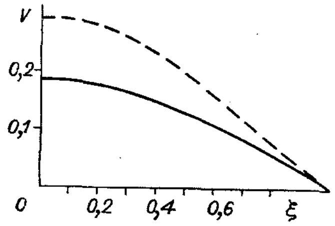

Fig. 5

Fig. 1 that with upward motion of the suspension in the given situation, the particles always tend to move toward the center of the channel – regardless of the ratio of the densities of the fluid and the disperse phase. This can be attributed to the fact that the Faxen force causes the particles to always lag behind the fluid. As a result, the transverse force is directed toward the center of the flow. The profiles of $\rho(\xi)$ in Fig. 3 were constructed for this case with $\Gamma = 100$. Here, the solid line pertains to $\alpha = 0.1$ and $\gamma = 0.2$, while the dashed line pertains to $\alpha = \gamma = 0$. This corresponds to the flow of an equidense suspension in the absence of a gravitational field. The profile in the figure coincides with the analogous profile from [5].

Since a change in the transverse coordinate ξ is accompanied by a change in the concentration of the disperse phase $\rho(\xi)$ for certain parameters α and γ, as the suspension moves downstram the absolute value of the Faxen force may become greater or less than the buoyant force minus the gravitational force. Thus, there is a change in the sign of phase-slip velocity U. This in turn leads to a change in the direction of the force (3). The profiles of concentration and velocity that develop in such a flow are shown in Figs. 4 and 5 (the solid lines correspond to $\Gamma = 100$, while the dashed lines correspond to $\Gamma = 250$). The parameters α and γ for this flow lie within the hatched region in Fig. 1.

As above, in using (9), we made up a table (Table 2) of values of concentration $\langle\rho\rangle$ and ρ_f. The calculations performed with $\alpha = \gamma = 0$ and $\Gamma = 100$ agree with the results reported in [4].

Thus, an explanation has been given for different types of structure-formation processes in vertical flows of Brownian suspensions in a gravitational field. It is apparent from the concentration profiles shown in Figs. 2-4 that such structuring is manifested to a greater degree with an increase in particle radius. This in turn causes the flow to begin to display pseudoplastic properties. Figure 5 describes experimental data which indicates that there is some smoothing of the velocity profiles and that the latter deviate from the Poiseuille profile in the presence of a nonuniform distribution of disperse-phase concentration.

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